

## • Multiplying Fractions and Whole Numbers

### Power Up

#### facts

#### mental math

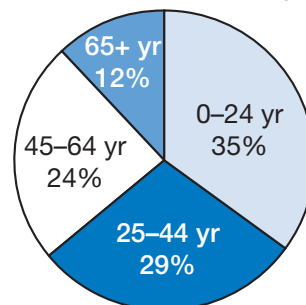
#### Power Up H

- Money:**  $3 \times (\$6 \text{ and } 25\text{¢})$
- Money:**  $5 \times (\$3 \text{ and } 25\text{¢})$
- Money:** One dollar is how many quarters?
- Measurement:** How many quarts are in a gallon?
- Powers/Roots:**  $8^2$
- Probability:** The sides of a number cube are labeled 1 through 6. If the cube is rolled once, what is the probability of rolling a number less than 5?
- Estimation:** Choose the more reasonable estimate for the mass of a hamster: 90 grams or 90 kilograms.
- Calculation:**  $\frac{1}{3}$  of 90,  $+ 3$ ,  $\div 3$ ,  $\times 9$

#### problem solving

Choose an appropriate problem-solving strategy to solve this problem. The circle graph at right is based on year 2004 estimates published by the Census Bureau. The graph shows the percentage of United States residents that belong to each of four age groups. Every 1 percent of the population is equal to nearly 3 million residents. Find the approximate number of U.S. residents who are 65 years of age or older.

U.S. Population by Age



### New Concept

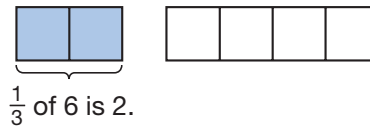
We have found a fraction of a whole number by dividing the whole number by the denominator of the fraction.

$$\frac{1}{3} \text{ of } 6 \text{ is } 2. \quad (6 \div 3 = 2)$$

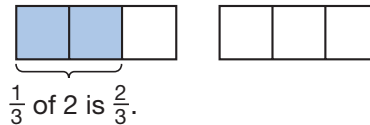
### Reading Math

When we multiply with fractions, the answer is stated in terms of the whole.

The model below illustrates that  $\frac{1}{3}$  of 6 rectangles is 2 rectangles.



How can we model  $\frac{1}{3}$  of 2? If we divide two whole rectangles into three parts each, then there are 6 parts in all, and  $\frac{1}{3}$  of 6 parts is 2 parts. We see that 2 parts is  $\frac{2}{3}$  of a whole rectangle.



An arithmetic method for finding  $\frac{1}{3}$  of 2 is to multiply.

What number is  $\frac{1}{3}$  of 2?

$$\begin{array}{c} \frac{1}{3} \text{ of } 2 \\ \downarrow \downarrow \downarrow \\ \frac{1}{3} \times \frac{2}{1} \end{array}$$

Notice that we wrote the whole number 2 as a fraction:  $\frac{2}{1}$ . Since 2 divided by 1 is 2, the fraction  $\frac{2}{1}$  equals 2. Writing the whole number as a fraction gives us a numerator and a denominator to multiply. The product is  $\frac{2}{3}$ .

$$\frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$$

Now we check for reasonableness. We know that  $\frac{1}{2}$  of 2 is 1. Since  $\frac{1}{3}$  is less than  $\frac{1}{2}$ ,  $\frac{1}{3}$  of 2 must be less than 1, and  $\frac{2}{3}$  is less than 1.

Here is another way to check our answer. Recall the Commutative Property of Multiplication. This property tells us that changing the order of factors does not affect the product. So another way to approach this problem is to switch the positions of  $\frac{1}{3}$  and 2.

$$\begin{array}{c} \frac{1}{3} \times 2 \\ \swarrow \searrow \\ 2 \times \frac{1}{3} \end{array} \quad \begin{array}{l} \text{We may reverse the order of} \\ \text{factors when we multiply.} \end{array}$$

Since  $2 \times \frac{1}{3}$  means  $\frac{1}{3} + \frac{1}{3}$ , we again find that the product is  $\frac{2}{3}$ .

## Example

**What number is  $\frac{2}{3}$  of 4?**

We know that  $\frac{2}{3}$  of 4 is greater than 2 because  $\frac{1}{2}$  of 4 is 2, and  $\frac{2}{3}$  is greater than  $\frac{1}{2}$ . We also know that  $\frac{2}{3}$  of 4 is less than 4. We multiply to find the answer.

$$\begin{array}{c} \frac{2}{3} \text{ of } 4 \\ \downarrow \downarrow \downarrow \\ \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3} \end{array}$$

We converted the improper fraction to a mixed number. Since  $2\frac{2}{3}$  is greater than 2 but less than 4, the answer is reasonable. We can check the answer by reversing the order of factors.

$$4 \times \frac{2}{3} \text{ means } \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$$

Again we get  $\frac{8}{3}$ , which equals  $2\frac{2}{3}$ .

## Lesson Practice

Multiply. Simplify answers when possible. Reverse the order of factors to check your answer.

a.  $\frac{1}{3} \times 4$

b.  $\frac{3}{5} \times 2$

c.  $\frac{2}{3} \times 2$

d. What number is  $\frac{1}{5}$  of 4?

e. What number is  $\frac{1}{6}$  of 5?

f. What number is  $\frac{2}{3}$  of 5?

g. **Model** Sketch rectangles to model  $\frac{1}{3}$  of 4. Begin by drawing four rectangles and then divide each rectangle into thirds. Then find  $\frac{1}{3}$  of the total number of parts.

## Written Practice

*Distributed and Integrated*

\*1. **Represent** Draw a pair of horizontal parallel segments. Make the lower segment longer than the upper segment. Make a quadrilateral by connecting the endpoints.

2. **Estimate** Find the difference between 6970 and 3047 by rounding the numbers to the nearest thousand and then subtracting.

3. **Represent** Write the following sentence using digits and symbols:

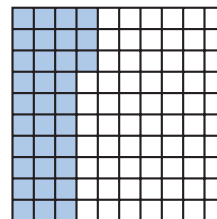
*The sum of six and four is ten.*

- \*4. A 2-liter bottle of water contains how many milliliters of liquid?

(85)

5. Name the shaded portion of this square as a fraction, as a decimal number, and as a percent:

(71)



- \*6. a. What number is  $\frac{1}{3}$  of 120?

(86)

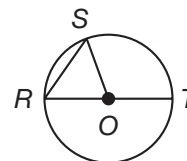
- b. What number is  $\frac{2}{3}$  of 120?

7. **Multiple Choice** Which segment names a diameter of this circle?

(53, 61)

- A  $\overline{RS}$   
C  $\overline{OS}$

- B  $\overline{RT}$   
D  $\overline{OT}$



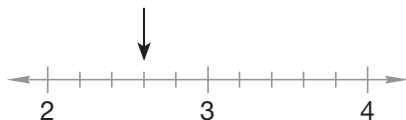
8. **List** Write these fractions in order from least to greatest:

(59, 75)

$$\frac{9}{18}, \frac{8}{7}, \frac{7}{16}, \frac{6}{6}, \frac{5}{8}$$

9. **Connect** To what mixed number is the arrow pointing?

(38)



Multiply to find each product in problems 10 and 11. Then reverse the order of factors to check your answers.

\*10.  $\frac{2}{3} \times 2$

(86)

\*11.  $\frac{3}{4}$  of 4

(86)

12.  $3 - \left(2\frac{3}{5} - 1\frac{1}{5}\right)$

(41, 63)

13.  $4.7 + 3.63 + 2.0$

(73)

$$\begin{array}{r} 14. \quad 301.4 \\ (73) \quad - 143.5 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 476 \\ (56) \quad \times 890 \\ \hline \end{array}$$

$$16. \quad 4 \overline{)348}$$

(26)

$$17. \quad 40 \overline{)3480}$$

(54)

$$18. \quad \$42.36 \div 6$$

(34)

$$19. \quad 22^2$$

(78)

- \*20. a. What are the common factors of 60 and 100?  
(82)

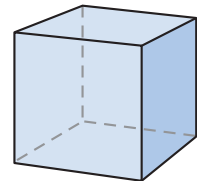
b. Use the GCF of 60 and 100 to reduce  $\frac{60}{100}$ .

- \*21. Write a fraction equal to  $\frac{3}{4}$  that has a denominator of 12. Then write a fraction equal to  $\frac{2}{3}$  that has a denominator of 12. Subtract the second fraction from the first fraction.  
(79)

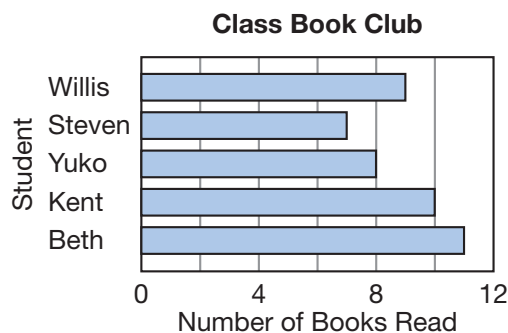
22. Since  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$ , how many  $\frac{3}{4}$ s are in  $\frac{9}{4}$ ?  
(Inv. 2)

- \*23. a. What is the name of this solid?  
(83)

b. How many vertices does it have?




- \*24. **Interpret** Use the graph below to answer parts a–c.  
(Inv. 7, 84)



- a. How many more books must Steven read to reach the goal of 12 books?
- b. Each book must have 180 pages or more. Kent has read at least how many pages so far?
- c. What is the median number of books read by the five students?
- \*25. What is the probability of rolling a number less than five with one toss of a standard number cube? Write the probability as a reduced fraction.  
(57, 81)

\*26. A quart is called a quart because it is a quarter of a gallon. What percent of a gallon is a quart?  
(71, 85)

\*27. Compare: 1 quart ○ 1 liter  
(85)

\*28.  **Explain** For exercise, Jia walks  $1\frac{1}{2}$  miles each morning and  $2\frac{1}{2}$  miles each evening. At that rate, how many days will it take Jia to walk 100 miles? Explain how you found your answer.  
(49, 75)

\*29. Refer to the following sequence to answer parts **a** and **b**.  
(Inv. 8)

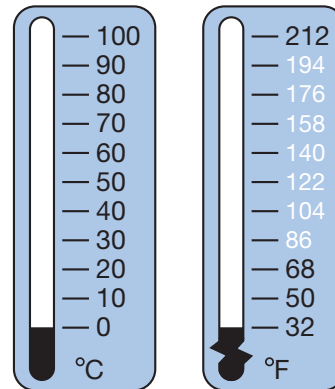


a. What is the next term in the sequence? Draw the answer on your paper.

b. **Multiple Choice** Which transformation describes the change from term to term?

- A** translation      **B** rotation      **C** reflection      **D** slide

\*30. A difference of  $100^\circ$  on the Celsius scale is a difference of  $180^\circ$  on the Fahrenheit scale. A  $10^\circ$  change on the Celsius scale is an  $18^\circ$  change on the Fahrenheit scale. Copy this thermometer on your paper, and label the remaining tick marks on the Fahrenheit scale.  
(27, Inv. 4)



### Early Finishers

Real-World Connection

Cherise is baking a cake. She wants to make a smaller cake that is  $\frac{2}{3}$  the size of the original recipe. If 3 cups of flour and 2 cups of milk are needed to make the cake in the recipe, how much flour and milk will she need to make the smaller cake?