

• Comparing Fractions by Drawing Pictures

Power Up

facts

Power Up F

count aloud

Count by 7s from 7 to 84.

mental math

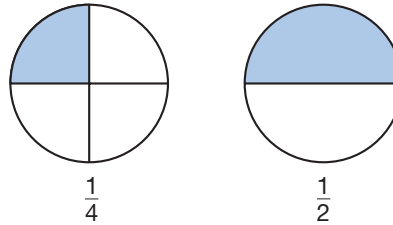
- a. **Time:** How many days is 3 weeks? ... 4 weeks?
... 6 weeks?
- b. **Estimation:** Round 78 to the nearest ten. Then add 70.
What is the answer?
- c. **Number Sense:** Apria had 830 pennies in her piggy bank.
She put 200 of the pennies into coin rolls. How many
pennies are left in her piggy bank?
- d. **Time:** How many hours is 6 days? (*Think:* 6×24 .)
- e. **Fractional Parts:** $\frac{1}{2}$ of 44 pounds
- f. **Percent:** 50% of \$2.00
- g. **Percent:** 25% of \$2.00
- h. **Calculation:** $5 \times 5, + 5, \div 5, - 5$

problem solving

Choose an appropriate problem-solving strategy to solve this problem. Lu'Ann is working through a book of 40 crossword puzzles. The puzzles are divided into five units according to difficulty, and each unit contains 8 puzzles. Lu'Ann began with the first puzzle of the book and is solving each puzzle in order until she finishes the book. She completes 6 puzzles each day. On which day will Lu'Ann's sixth puzzle for the day be the final puzzle in one of the book units? How many puzzles will she have completed at that point?

New Concept

One fourth of the circle on the left is shaded. One half of the circle on the right is shaded.



We see that less of the circle on the left is shaded. This is because $\frac{1}{4}$ is a smaller fraction than $\frac{1}{2}$. We can write this comparison using a comparison symbol as

$$\frac{1}{4} < \frac{1}{2}$$

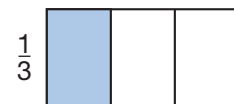
In this lesson we will begin comparing fractions by drawing pictures of the fractions and comparing the pictures.

Example

Armando finished his math assignment in $\frac{1}{3}$ of an hour. Jan finished the assignment in $\frac{1}{2}$ of an hour. Who took longer to finish the assignment? Draw pictures to compare these fractions: $\frac{1}{2} \bigcirc \frac{1}{3}$

We might think that $\frac{1}{3}$ is greater than $\frac{1}{2}$ because 3 is greater than 2. However, by drawing pictures, we will see that $\frac{1}{3}$ is actually less than $\frac{1}{2}$. If an object is divided into 3 parts, each part will be smaller than if the object were divided into 2 parts.

To begin, we draw two *congruent* shapes. We draw two equal-sized rectangles, and we label the rectangles $\frac{1}{2}$ and $\frac{1}{3}$. Next, we divide the rectangles into the number of parts shown by the denominator, and we shade the number of parts shown by the numerator.



Then we compare the shaded areas. We see that more of the rectangle is shaded when $\frac{1}{2}$ is shaded than when $\frac{1}{3}$ is shaded.

$$\frac{1}{2} > \frac{1}{3}$$

Since $\frac{1}{2}$ is greater than $\frac{1}{3}$, $\frac{1}{2}$ of an hour is greater than $\frac{1}{3}$ of an hour, so **Jan took longer** to finish the assignment.

Generalize These fractions are written in order from least to greatest:

$$\frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

What conclusion can be made about the values of fractions when the numerators are the same?

Lesson Practice

Represent Draw pictures to compare each pair of fractions. When drawing pictures of any two fractions, be sure to draw the shapes the same size.

a. $\frac{1}{2} \bigcirc \frac{2}{3}$

b. $\frac{1}{2} \bigcirc \frac{2}{4}$

c. $\frac{1}{3} \bigcirc \frac{1}{4}$

d. $\frac{2}{3} \bigcirc \frac{3}{4}$

Written Practice

Distributed and Integrated

* 1. **Represent** Draw a pair of horizontal parallel line segments of the same length. Form a quadrilateral by connecting the ends of the segments.
(31, 32)

Formulate For problems 2–4, write an equation and find the answer.

2. How many years is five centuries?
(21, 28)

3. Paloma is 6 years older than her sister. If Paloma is 13 years old, then how old is her sister? Use a subtraction pattern.
(35)

4. Diego walked 488 feet going to the end of the pier and back. How long is the pier?
(21)

5. **Represent** Draw pictures to compare these fractions: $\frac{1}{4} \bigcirc \frac{1}{3}$
(39)

6. What number is half of 23?
(2)

7. Emily's cat ate $\frac{1}{4}$ of a dozen fish fillets. How many fish fillets did Emily's cat eat?
(Inv. 2)

8. Round 84 to the nearest ten.
(33)

- *9. List** Write the factors of 35.
(25)

$$\begin{array}{r} 10. \quad \$93.18 \\ (13) \quad \$42.87 \\ + \$67.95 \\ \hline \end{array}$$

$$\begin{array}{r} 11. \quad \$30.00^* \\ (13) \quad - \$ 8.75 \\ \hline \end{array}$$

$$\begin{array}{r} *12. \quad 46 \\ (6) \quad 23 \\ 97 \\ 15 \\ 24 \\ 55 \\ + 55 \\ \hline \end{array}$$

$$\begin{array}{r} 13. \quad 4304 \\ (14) \quad - \quad b \\ \hline 3452 \end{array}$$

$$\begin{array}{r} 14. \quad \$6.38 \\ (29) \quad \times \quad 60 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad 640 \\ (29) \quad \times \quad 700 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 640 \\ (34) \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 17. \quad 720 \\ (26) \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} 18. \quad \$6.24 \\ (34) \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 19. \quad 1236 \\ (34) \quad 4 \\ \hline \end{array}$$

20. $563 \div 7$
(34)

21. $4718 \div 9$
(26)

22. $8m = 3000$
(26)

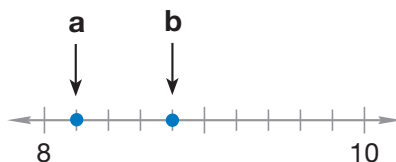
23. The latest showing of a movie ends at 20 minutes before midnight.
(28) At what time does the movie end?

- *24. A quarter of a circle plus an eighth of a circle is what percent of a circle?
(Inv. 2, Inv. 3)

25. According to this calendar, what was the date of the third Saturday in April 1901?
(28)

APRIL 1901						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8		10	11	12	13
14	15	16	17	18	1	20
21	22	23	24	25	26	27
28	2	30				

- *26. **Connect** Refer to the number line below to answer parts a–c.
(38)



- a. To what mixed number is arrow **a** pointing?
- b. To what mixed number is arrow **b** pointing?
- c. Write your answers to **a** and **b** using a comparison symbol to show which mixed number is greater and which is less.


27. What is the product of four hundred sixteen and sixty?
(5, 29)

28. a. How many hours are in a day?
(23, 28)

b. How many hours are in half a day?

c. Use your answers to parts **a** and **b** to write a fraction equal to $\frac{1}{2}$.

29. D'Arla works each day from 7:45 a.m. to 4:15 p.m. During that time, she is not paid for a 45-minute lunch break. How many hours is D'Arla paid for each day she works?
(35)

30.  **Estimate** A group of friends are planning to spend 7 days hiking a 135-mile portion of the Appalachian Trail and expect to hike about the same number of miles each day. Estimate the number of miles the friends plan to hike each day. Explain your answer.
(33)

Early Finishers

Real-World Connection

Seth and Margie each ordered a 12-slice veggie pizza for dinner. Seth ate $\frac{1}{2}$ of his pizza. Margie ate $\frac{1}{3}$ of her pizza.

- Draw two circles. Then shade one circle to represent the amount of pizza that Seth ate, and shade one circle to represent the amount of pizza that Margie ate.
- Which is greater: $\frac{1}{2}$ or $\frac{1}{3}$? Write a comparison using the fractions and a comparison symbol.
- Find the number of slices of pizza Seth ate and the number of slices that Margie ate.

• Writing Quotients with Mixed Numbers

Power Up

facts

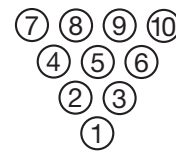
Power Up E

mental math

- Time:** What time is 22 minutes after 3:15 p.m.?
- Number Sense:** 600×4
- Measurement:** Twelve inches is one foot. How many inches is half a foot? ... one and a half feet? ... two and a half feet?
- Fractional Parts:** $\frac{1}{2}$ of 21
- Money:** $10 \times 100\text{¢}$
- Percent:** What is 50% of \$20.00?
- Percent:** The shirt is on sale for 25% off the regular price of \$20.00. How much is 25% of \$20.00?
- Calculation:** $4 \times 2, + 1, \times 3, + 3, \div 3, \div 2$

problem solving

In bowling, a *spare* occurs when two rolls are used to knock down all 10 pins. Knocking down 3 pins on the first roll and 7 pins on the second roll is one way to bowl a spare. What are all the possible ways to get a spare in bowling?



Focus Strategy: Make or Use a Table, Chart, or Graph

Understand We are told that a spare in bowling occurs when two rolls are used to knock down all 10 pins. We are given an example of one of the ways to make a spare. We are asked to find all the possible ways to get a spare.

Plan We will make a table to help us find all the possible ways to make a spare. A table will help us organize our answer and ensure that we find all the possible ways to make a spare.

Solve We create two columns for our table and label the columns “1st roll” and “2nd roll.” We start filling in the table by writing what happens if 0 pins are knocked down on the first roll. We write “0” in the first column. For a spare to occur, all ten pins must be knocked down, so 0 on the first roll means 10 pins are knocked down on the second roll. We write “10” in the second column.

Next, we record what happens when 1 pin is knocked down on the first roll. Nine pins would remain to be knocked down on the second roll, so we write “1” in the first column and “9” in the second column.

We continue to add rows to the table until we reach 9 pins on the first roll and 1 pin on the second roll.

1st roll	2nd roll
0	10
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1

Check We know that our answer is reasonable because we found two numbers that total 10 for each possible first roll of the ball. We did not include a row in the table for 10 pins on the first roll. Knocking down 10 pins on the first roll—this called a *strike*—means that all the pins are knocked down, and there is no need for a second roll. Making a table helped us find all the possibilities.

New Concept

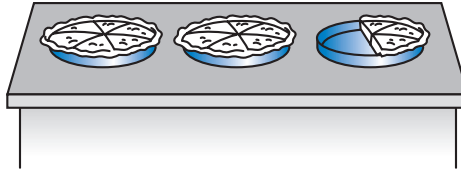
Reading Math

A mixed number can represent the sum of whole numbers and a fraction.

$$1 + 1 + \frac{1}{2} =$$

$$2 + \frac{1}{2} = 2\frac{1}{2}$$

The picture below shows some potpies on a shelf.

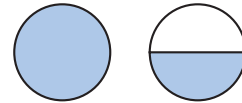


We see two whole potpies and one half of another potpie. There are two and one half potpies on the shelf. Using digits, we write “two and one half” this way:

$$2\frac{1}{2}$$

Example 1

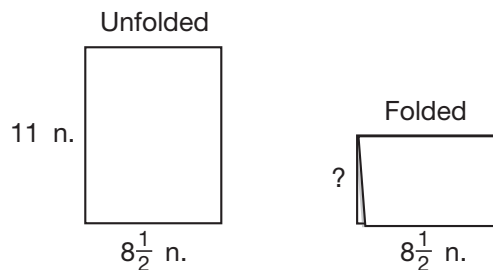
Use a mixed number to name the number of shaded circles shown here.



We see two circles. The completely shaded circle represents the whole number 1. Half of the second circle is shaded. It represents the fraction $\frac{1}{2}$. Together, the number of shaded circles is one and one half.

$$1\frac{1}{2}$$

Some problems have answers that are mixed numbers. For example, what is the width of the rectangle formed by folding a sheet of notebook paper in half?

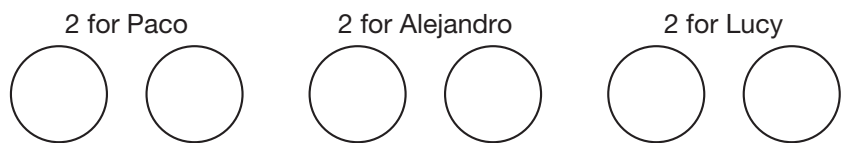


The width of the rectangle is half of 11 inches, which is $5\frac{1}{2}$ inches.

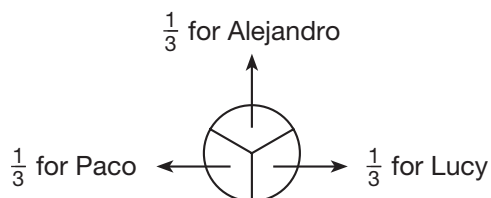
Example 2

Paco, Alejandro, and Lucy will equally share seven chicken potpies. How many pies are there for each person?

First we will use a diagram to explain the solution. We need to divide the potpies into three equal groups. We can arrange six of the potpies into three groups of two potpies.



However, there are seven potpies, so there is still one potpie to be divided. We divide the remaining potpie into thirds:



We find that there are $2\frac{1}{3}$ potpies for each person.

Now we will show how to find the answer using a pencil-and-paper algorithm. To divide seven potpies into three equal groups, we divide 7 by 3.

$$\begin{array}{r} 2 \\ 3 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

The quotient is 2, which means “2 whole potpies.” The remainder is 1, which means one potpie has not been divided. Now we divide the remaining potpie by three.

One divided by three is the fraction one third. We write “ $\frac{1}{3}$ ” after the whole number above the division box.

$$\begin{array}{r} 2\frac{1}{3} \\ 3 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

This answer means that each person will get $2\frac{1}{3}$ potpies.

Math Language

The remainder of a division problem is written as the numerator of a fraction that has the divisor as its denominator.

Example 3

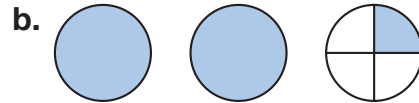
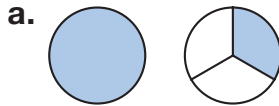
A grocer divided a 5-pound block of mozzarella cheese into 4 equal pieces. The pieces were then wrapped and displayed for sale. Which quotient represents the weight in pounds of each piece?

$$\begin{array}{r} 1 \text{ R } 1 \\ 4 \overline{)5} \\ -4 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1\frac{1}{4} \\ 4 \overline{)5} \\ -4 \\ \hline 1 \end{array}$$

It is possible to divide the cheese into 4 pieces of equal weight, so the weight of each piece is $1\frac{1}{4}$ pounds.

Lesson Practice

Write a mixed number to name the number of shaded circles in each diagram:



Represent Draw and shade circles to represent each mixed number:

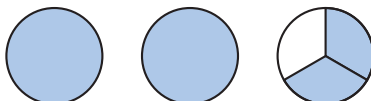
- c. three and one half
- d. one and three fourths
- e. **Represent** Use a diagram to explain the answer to the problem below. Then show how to find the answer using pencil and paper.

Taro, Shasa, Layne, and Cynthia will equally share nine chicken potpies. How many pies are there for each person?

Written Practice

Distributed and Integrated

- * 1. **Represent** Draw a pair of horizontal parallel line segments. Make the lower segment longer than the upper segment. Connect the endpoints of the segments to form a quadrilateral.
(31, 32)
- 2. If 1 pizza is shared equally by 6 people, then each person will get what fraction of the pizza?
(37)
- 3. Hikaru, Luz, and Obi are sharing 4 oranges equally. How many oranges does each person have?
(40)
- 4. How many circles are shaded?
(40)



Formulate For problems 5–7, write an equation and find the answer.

*5. ⁽²¹⁾ One hundred forty students were divided equally into 5 classes. How many students were in each class?

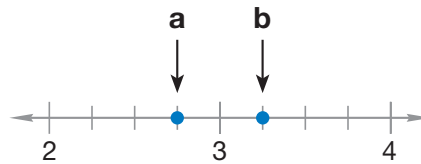
6. ⁽³⁵⁾ A veterinarian measured the weight of Khanh's two golden retrievers as 69 pounds and 83 pounds. How many fewer pounds did Khanh's 69-pound golden retriever weigh?

7. ⁽³⁵⁾ The first flag of the United States had 13 stars. How many more stars does the current flag have?

8. ⁽³²⁾ A hexagon has how many more sides than a pentagon?

9. ^(Inv. 2) One half of a circle plus one fourth of a circle is what percent of a whole circle?

*10. **Connect** ⁽³⁸⁾ Refer to the number line below to answer parts a–c.



a. To what mixed number is arrow **a** pointing?

b. To what mixed number is arrow **b** pointing?

c. Write your answers to **a** and **b** using a comparison symbol to show which number is greater and which is less.

11. ^(2, Inv. 3) What percent is half of 25%?

12. ⁽¹⁴⁾ $m - 345 = 534$

$$\begin{array}{r} 13. \quad 785 \\ \quad 964 \\ \quad 287 \\ + 846 \\ \hline \end{array}$$

$$\begin{array}{r} 14. \quad 7106 \\ \quad - 3754 \\ \hline \end{array}$$

$$\begin{array}{r} 15. \quad \$3.84 \\ \quad \times \quad 60 \\ \hline \end{array}$$

16. ⁽²⁹⁾ 769×800

$$17. \quad \frac{\$24.48}{8}$$

$$18. \quad \frac{4320}{9}$$

19. ^(13, 24) $\$20 - (\$1.45 + \$6.23 + \$8)$

20. ^(13, 17) $3742 + 3742 + 3742 + 3742 + 3742$

21. Round 650 to the nearest hundred.
(33)

22. A year is what fraction of a decade? A year is what percent of a decade?
(28)

23. **Multiple Choice** Which of these angles appears to be an obtuse angle?
(31)



24. **Conclude** What are the next three terms in this counting sequence?
(1)

..., 60, 70, 80, _____, _____, _____, ...

25. On April 2, the low temperature of the day in Madison, Wisconsin, was 48°F. The high temperature of the day was 13°F higher. What was the high temperature that day?
(27)


26. **Represent** Draw two circles of the same size. Shade $\frac{1}{4}$ of one circle and $\frac{1}{3}$ of the other circle. Then compare these fractions:
(39)


$$\frac{1}{4} \bigcirc \frac{1}{3}$$

* 27. **Multiple Choice** Which of these fractions is greater than one half?
(23)

A $\frac{5}{12}$ B $\frac{3}{5}$ C $\frac{7}{14}$ D $\frac{10}{21}$

28. At a bus stop, the first three scheduled bus arrival times of the day are 6:42 a.m., 7:17 a.m., and 7:52 a.m. If that pattern continues, what is the next scheduled arrival time for the bus?
(35)

29.  **Estimate** A marathon is a long-distance running event. Steve has run in four Boston Marathons. The distance of each marathon was 26 miles 385 yards. Explain how to estimate the total number of miles Steve has run in Boston Marathons.
(1)

30.  **Estimate** A soccer coach conducts 6 different drills during the first 55 minutes of each soccer practice. Explain how to estimate the length in minutes of each drill if the coach spends about the same number of minutes conducting each drill.
(33)